MUST Journal of Research and Development (MJRD) Volume 6 Issue 3, September 2025
e ISSN 2683-6467 & p ISSN 2683-6475

Critical Review of the Percentage of Cumulative Oil Production with Sequential Quadratic Programming Technique for Gas Lifted Wells

[1]Isaac Eze Ihua-Maduenyi*, [2]Zekieni Robert Yelebe and [3]Sunday Igbani
[1]Department of Petroleum Engineering, Faculty of Engineering, P.M.B. 5080, Port Harcourt, Rivers State University, Nigeria
[2,]Department of Chemical Engineering, Faculty of Engineering, Niger Delta University, Wilberforce Island, P.M.B. 71, Bayelsa State, Nigeria
[3]Department of Petroleum Engineering, Faculty of Engineering, Niger Delta University, Wilberforce Island, P.M.B. 71, Bayelsa State, Nigeria
DOI: https://doi.org/10.62277/mjrd2025v6i30005

ARTICLE INFORMATION

Article History

Received: 18th June 2025
Revised: 26th August 2025
Accepted: 02nd September 2025
Published: 30th September 2025

Keywords

Sequential Quadratic Programming
Gas Lift, Optimisation
Lagrangian
Cummulative Oil Production

ABSTRACT

Gas lift optimisation presents a complex, nonlinear constrained problem in petroleum engineering, where dynamic well interactions, multiphase flow behaviour, and stringent operational constraints pose significant computational challenges. This study systematically reviews the application of Sequential Quadratic Programming (SQP) as an advanced numerical optimisation technique for gas lift performance enhancement. SQP's mathematical foundation, rooted in second-order approximations of the objective function and constraints, leverages Hessian approximations and Lagrange multipliers to achieve superior solution accuracy and convergence efficiency. Comparative analyses demonstrate SQP's superiority over conventional optimisation methods such as Mixed-Integer Linear Programming (MILP) and the Augmented Lagrangian (AL) method. Unlike MILP, which struggles with nonlinear deliverability constraints, and AL, which exhibits minor constraint violations, SQP ensures strict constraint adherence while optimising gas injection rates. The method's computational efficiency is attributed to advanced gradient estimation, parallel processing capabilities, and QR factorisation updates, making it highly effective for large-scale gas lift networks. Notably, SQP-driven optimisation has been shown to improve Net Present Value (NPV) by up to 42% and increase oil production by 45% through optimal gas allocation and stabilisation of intermittent flow regimes. Furthermore, the adaptability of SQP for real-time optimisation enables its seamless integration into industry-standard production simulation tools such as PROSPER, GAP, and OLGA, facilitating dynamic field-wide gas lift coordination. Emerging hybrid SQP frameworks, incorporating augmented Lagrangian strategies and nonlinear steady-state optimisation, further enhance solution robustness and economic performance. Crucially, SQP's ability to model real-world constraints—including reservoir pressure limits, gas-lift performance curves, and fluctuating operational conditions—demonstrates its viability for practical implementation in complex petroleum production systems. This review establishes SQP as a transformative optimisation framework for gas lift operations, bridging theoretical advancements with real-world applicability. The findings underscore SQP's computational and economic advantages over conventional methods while paving the way for future research into hybridised algorithms and real-time adaptive gas lift control in large-scale petroleum production networks.

*Corresponding author's e-mail address: isaac.ihua-maduenyi@ust.edu.ng (Ihua-Maduenyi, I.Z)

1.0 Introduction

Gas lift optimisation is a crucial aspect of production engineering that aims to enhance oil recovery from wells by efficiently injecting gas to reduce fluid density and improve lift performance. Given the increasing complexity of oil and gas reservoirs, optimising gas lift operations is essential for maximising production while minimising operational costs (Abdalsadig *et al.,* 2016). Traditional gas lift optimisation techniques often rely on heuristic methods, empirical correlations, or simple rule-based approaches. While these methods offer some degree of efficiency, they lack the robustness and adaptability required to handle the nonlinear, constrained, and multi-variable nature of gas lift systems. Consequently, numerical optimisation techniques have gained prominence in recent years, providing more systematic and mathematically rigorous approaches to solving gas lift problems (Pacheco *et al.,* 2023).

Numerical optimisation methods provide systematic frameworks for solving gas lift optimisation problems by identifying the optimal injection gas rates and lift point locations. Various algorithms have been proposed to address these challenges, ranging from classical mathematical optimisation techniques to modern machine learning-based approaches. Among numerical optimisation methods, Sequential Quadratic Programming (SQP) has emerged as one of the most effective approaches for handling constrained nonlinear optimisation problems such as gas lift allocation (Ihua-Maduenyi & Oguta, 2025; Salehian *et al.*, 2021). SQP is a gradient-based method that iteratively solves a sequence of quadratic programming subproblems to approximate the solution of a nonlinear programming problem. It efficiently handles equality and inequality constraints while ensuring convergence to a local optimum. The application of SQP to gas lift optimisation is particularly attractive because gas lift operations involve complex physical models, including multiphase flow behaviour, pressure-volume-temperature (PVT) relationships, and wellbore hydraulics, which introduce strong nonlinearities and constraints that must be satisfied for practical feasibility (Noorbakhsh & Khamehchi, 2020).

One of the key advantages of SQP in gas lift optimisation is its ability to systematically account for both reservoir and operational constraints. Unlike simpler optimisation methods such as gradient descent or genetic algorithms, SQP explicitly incorporates second-order information through the Hessian matrix, allowing for more accurate search directions and faster convergence rates (Al-Mansory *et al.,* 2024). This feature is particularly beneficial in gas lift systems, where the response surface is often highly nonlinear, and an efficient optimisation algorithm must navigate complex feasibility regions to achieve optimal gas allocation. Additionally, SQP's robustness in handling inequality constraints ensures that operational limits, such as compressor capacity, tubing pressure, and gas injection rates, are consistently respected throughout the optimisation process (Zhong *et al.,* 2022).

Comparatively, heuristic-based methods such as trial-and-error or rule-based optimisation approaches are limited by their dependence on empirical knowledge and lack of scalability when dealing with large-scale gas lift networks. While evolutionary algorithms like genetic algorithms (GA) and particle swarm optimisation (PSO) offer alternative optimisation strategies, they generally require a large number of function evaluations and lack the deterministic convergence guarantees that SQP provides. Metaheuristic approaches often struggle with constraint satisfaction, making them less reliable for applications where strict operational and engineering constraints must be maintained. In contrast, SQP efficiently converges to feasible and near-optimal solutions within a relatively small number of iterations, making it a preferred choice for gas lift optimisation (Vazquez-Roman & Palafox-Hernández, 2005).

Furthermore, the adaptability of SQP to different gas lift scenarios makes it particularly suitable for field-wide optimisation. Given that gas lift operations involve multiple wells with varying production characteristics, an optimisation approach must be capable of dynamically adjusting gas injection rates based on real-time production data (Sharma *et al.,* 2012). SQP, with its ability to integrate with real-time reservoir and well models, provides a structured framework for adaptive optimisation, ensuring that production targets are met while minimising gas usage. This adaptability also allows for seamless integration with digital oilfield technologies, such as production monitoring systems and artificial

intelligence-driven predictive models, further enhancing the efficiency and reliability of gas lift optimisation strategies (Tewari & Agrawal, 2022).
The implementation of SQP in gas lift optimisation also has significant economic and environmental implications. By ensuring optimal gas allocation, SQP minimises unnecessary gas consumption, leading to reduced operational costs and improved energy efficiency (Sreenivasan *et al.,* 2023). This is particularly important in mature fields where gas availability may be limited, and excessive gas injection can lead to diminishing returns in oil production. Moreover, efficient gas lift optimisation contributes to sustainability by reducing greenhouse gas emissions associated with gas compression and injection. As the oil and gas industry continues to prioritise operational efficiency and environmental responsibility, adopting advanced optimisation techniques such as SQP becomes increasingly valuable (Tavakoli *et al.,* 2017).

2.0 Gas Lift Optimisation

Gas lift optimisation is a crucial process in petroleum production engineering; it ensures that oil recovery is maximised while minimising operational costs. Gas lift optimisation primarily revolves around selecting the optimal volume of gas to inject into a set of wells to enhance oil production. Since lift gas is both a valuable and costly resource, excessive injection can lead to diminishing returns due to frictional constraints, while insufficient gas injection results in suboptimal production rates (Salehian *et al*., 2021).
The fundamental principle behind gas lift is that injecting gas into the production tubing reduces the hydrostatic pressure, thereby lowering the fluid density and allowing for an increased oil production rate. However, increasing the injection rate beyond an optimal point leads to excessive pressure drops due to frictional losses, which ultimately counteract the benefits of gas lift (Pacheco *et al.,* 2023). This relationship is captured in the Gas Lift Performance Curve (GLPC), a dome-shaped curve that illustrates how production rates vary with different gas injection rates. The GLPC highlights that under-injection leads to lower production, whereas excessive injection causes production decline and increased operational expenses (Okorocha *et al.,* 2020).
When optimising gas lift at the well level, various operational parameters must be considered, such as fluid composition, tubing geometry, pressure conditions, and completion type (Obong et al., 2022). A well-based approach typically involves conducting step-rate gas injection tests and analysing the data to determine the most effective injection strategy. Single-well optimisation focuses on modelling individual well behaviour, employing either black-oil models or more detailed computational fluid dynamics simulations to generate accurate lift performance curves (Okorocha et al., 2022). These curves help in determining the optimal gas injection rate for maximising production under given well conditions. Although nodal analysis techniques are widely used, they offer an incomplete field-wide solution because they don't capture interactions between wells.
A key challenge in well-based gas lift optimisation is the inability to capture interdependencies among multiple wells sharing a common lift gas supply. In multi-well systems, gas allocation must be strategically managed to ensure that gas is distributed effectively among the wells to achieve maximum overall production. Without proper optimisation, allocating excessive gas to certain wells can lead to suboptimal production from other wells due to backpressure effects and network constraints. The gas injection pressure and available lift gas volume impose additional limitations on well-based optimisation, necessitating a broader field-wide perspective (Okafor & Loyibo, 2024).
Field-based gas lift optimisation differs significantly from well-based optimisation in that it takes into account the entire network of interconnected wells and associated surface facilities. Unlike single-well optimisation, field-wide optimisation considers factors such as flowline pressure drops, compressor limitations, separator capacities, and water-handling constraints (Abdalsadig *et al.,* 2016). The complexity of field-based optimisation arises from the dynamic interactions between wells, where changes in the gas injection rate of one well can influence the pressure conditions and production performance of other wells within the network. Addressing these interdependencies is critical for

achieving an optimal allocation of lift gas that maximises total field production while adhering to facility constraints (Adukwu *et al*., 2023).

Several challenges complicate field-wide gas lift optimisation. Limited gas availability necessitates strategic allocation to ensure that the wells with the highest production potential receive an adequate gas supply. Additionally, backpressure effects caused by gas injection and production from interconnected wells can lead to suboptimal performance if not properly managed (Sreenivasan *et al*., 2024). Surface equipment constraints, such as compressor capacity and separator limitations, further restrict the total amount of gas that can be injected and the volume of produced fluids that can be handled. Moreover, unexpected issues such as well shut-ins and workovers introduce additional complexities that must be accounted for in the optimisation process (Hannanu *et al*., 2024).

Optimisation methods for gas lift allocation can be broadly categorised into numerical and heuristic approaches. Numerical methods involve mathematical optimisation techniques such as linear programming, nonlinear programming, and dynamic programming (Agwu *et al*., 2024). These methods rely on well-defined objective functions and constraints to determine the optimal gas allocation strategy. Sequential Quadratic Programming (SQP) is one of the most effective numerical techniques used in gas lift optimisation, as it efficiently handles nonlinear constraints and provides accurate solutions for complex multi-well optimisation problems (Ihua-Maduenyi & Oguta, 2025). SQP iteratively approximates the solution using a series of quadratic subproblems, making it particularly useful for optimising large-scale field networks where interactions between wells must be considered (Rostamian *et al*., 2024).

Heuristic optimisation methods, such as genetic algorithms, particle swarm optimisation, and simulated annealing, have also been widely applied to gas lift optimisation problems (Avriel, 2020). These methods are particularly beneficial when dealing with highly complex and nonlinear field models where traditional numerical techniques struggle to converge to a global optimum. Heuristic approaches explore a broad solution space using adaptive search strategies, making them suitable for optimising gas allocation under uncertainty and varying operational conditions. However, these methods require significant computational effort and may not always guarantee globally optimal solutions (Jonatian, 2024).

Incorporating real-time data analytics and artificial intelligence (AI) has further enhanced gas lift optimisation efforts (Ihua-Maduenyi & Yelebe, 2025). Machine learning models can analyse historical production data and predict optimal gas injection rates based on real-time well conditions. By integrating AI-driven optimisation techniques with advanced reservoir simulation models, operators can dynamically adjust gas lift allocation strategies to maximise production efficiency (Zeinilabedini & Ameli, 2025). Digital twins, which create virtual representations of field operations, enable real-time monitoring and optimisation by simulating different gas injection scenarios and identifying the best course of action based on current field conditions.

Despite the advancements in gas lift optimisation methodologies, several challenges remain. Accurate modelling of well and field performance requires high-quality data, which is not always readily available. Variations in reservoir conditions, fluid properties, and equipment performance introduce uncertainties that must be accounted for in optimisation models (Ahmed *et al.,* 2023). Additionally, implementing optimisation recommendations in real-world operations requires seamless integration with existing production management systems and control infrastructure.

The comparison of well-based and field-based gas lift optimisations is presented in Table 1.

Table 1
Comparison of Well-Based and Field-Based Gas Lift Optimisations

Aspect	Well-Based Optimisation	Field-Based Optimisation
Focus	Individual well performance	Entire field network performance
Key Parameters	Injection rate, depth, tubing diameter, pressure	Gas allocation, backpressure, separator limits
Complexity	Lower complexity, localized analysis	Higher complexity, requires network modeling
Interaction Effects	Does not consider inter-well dependencies	Accounts for interactions among wells
Optimisation Scope	Maximising single-well output	Maximising total field production
Constraints	Well-level constraints (e.g., tubing size)	Field-wide constraints (e.g., gas availability)

3.0 Sequential Quadratic Programming

3.1 History of Sequential Quadratic Programming

The Sequential Quadratic Programming (SQP) method has a rich history in the field of numerical optimisation, tracing back to the 1960s and 1970s. The method emerged as a powerful technique for solving nonlinear constrained optimisation problems, building upon principles of quadratic approximation and iterative refinement. SQP evolved as an extension of Newton's method applied to constrained optimisation, aiming to iteratively approximate and solve a sequence of quadratic programming subproblems. Early developments were driven by the need for efficient numerical solutions in mathematical programming, with pioneering contributions from researchers such as Wilson (1963) and Han (1977), who formalised the use of quadratic programming in constrained optimisation. Powell (1978) and Boggs & Tolle (1995) further refined the technique, introducing robust algorithms for handling both equality and inequality constraints (Bomze *et al.,* 2010).

As SQP matured, it found widespread applications across engineering disciplines, where optimisation problems are often nonlinear and constrained by complex physical and operational constraints (Noorbakhsh and Khamehchi, 2020). In mechanical engineering, SQP has been extensively used in structural optimisation, aerodynamics, and robotics, where precise control over parameters is essential (Krishnamoorthy *et al.,* 2019). The aerospace industry has leveraged SQP for trajectory optimisation, flight control, and spacecraft guidance, benefiting from its ability to efficiently handle multiple nonlinear constraints. Similarly, in electrical and control engineering, SQP has been applied to optimal control problems, power system optimisation, and signal processing, demonstrating its versatility and effectiveness in handling large-scale engineering challenges (Bandekian, 2023).

Sequential Quadratic Programming (SQP) has played a crucial role in petroleum engineering, particularly in reservoir management, production planning, and artificial lift systems. The ability of SQP to handle nonlinear optimisation problems with constraints has made it a preferred choice for tackling complex engineering challenges (Krishnamoorthy *et al.,* 2018). In petroleum engineering, it has been extensively utilised for optimising well placement, refining reservoir simulation models, and improving production strategies by determining the most effective operating conditions. Its flexibility and efficiency in solving multi-variable and multi-constraint problems make it indispensable in the field (Krishnamoorthy *et al.,* 2019).

A particularly significant application of SQP in petroleum engineering is in gas lift optimisation, where precise control of gas injection rates is crucial for maximising oil production while minimising operational costs. Gas lift, an artificial lift technique, relies on injecting gas into the wellbore to reduce hydrostatic pressure and enhance oil flow (Liu *et al.*, 2018). Traditional optimisation methods, such as nodal analysis, have been effective for single-well scenarios but struggle with the complexity of interconnected gas-lifted field networks. SQP overcomes this limitation by incorporating nonlinear constraints related to fluid flow dynamics, gas allocation, and facility constraints. It enables production engineers to determine the optimal gas injection rates for each well in a network while accounting for backpressure effects and well interdependencies (Lu *et al.*, 2016).

The use of SQP in gas lift optimisation involves modelling the relationship between gas injection rates and oil production while considering factors such as well interactions, backpressure, and compressor limitations. The iterative structure of SQP facilitates continuous adjustments to gas injection strategies, ensuring solutions remain feasible within operational constraints. Moreover, SQP has been integrated into real-time optimisation frameworks, working alongside production monitoring systems to dynamically adapt gas lift parameters based on evolving reservoir conditions. This adaptability enhances both the efficiency and effectiveness of gas lift operations (Bomze, 2010; Curtis *et al.*, 2012).

Researchers have extensively explored SQP-based gas lift optimisation models to improve gas allocation strategies across multi-well systems. These models aim to maximise overall field production by distributing the available lift gas in an optimal manner. To enhance computational efficiency, hybrid approaches have emerged, combining SQP with heuristic optimisation techniques such as genetic algorithms and particle swarm optimisation. These hybrid methodologies leverage the robustness of SQP in handling constraints while utilising heuristic methods to explore a broader solution space efficiently (Ihua-Maduenyi & Oguta, 2025; Mehregan *et al.,* 2016). The practical effectiveness of SQP in gas lift optimisation has been demonstrated through numerous field applications and case studies. Studies have consistently shown that SQP-driven optimisation can significantly enhance oil recovery, leading to substantial financial benefits for operators. The integration of SQP into commercial production optimisation software allows engineers to systematically analyse and refine gas injection strategies, ensuring long-term sustainability and operational efficiency. Additionally, advancements in computational power and real-time data integration continue to improve the robustness and applicability of SQP in gas lift systems, solidifying its position as a critical tool in modern petroleum engineering (Curtis *et al.,* 2012).

Despite its advantages, SQP faces challenges when applied to large-scale gas lift optimisation problems. The computational cost of solving successive quadratic subproblems can be high, especially when dealing with high-dimensional optimisation models. To address this, researchers have introduced modifications to the SQP algorithm, such as employing surrogate models and reduced-order approximations to expedite computations. Furthermore, advancements in parallel computing and machine learning techniques have been leveraged to enhance the scalability of SQP, making real-time gas lift optimisation more feasible (Bomze *et al.,* 2010).

As the petroleum industry continues to embrace digitalisation and automation, the role of SQP in gas lift optimisation will become even more prominent. The integration of artificial intelligence, real-time data analytics, and cloud computing with SQP-based optimisation frameworks will enable faster decision-making and improved production performance. These advancements will ensure that gas lift operations remain cost-effective, environmentally sustainable, and technically efficient, further cementing SQP's importance in petroleum engineering and beyond (Bomze *et al.*, 2010).

3.2 Theoretical Foundation of SQP

The principle of SQP is based on the iterative solution of a sequence of Quadratic Programming (QP) subproblems that approximate the original nonlinear constrained optimisation problem. It combines quadratic approximation techniques with Lagrange multiplier updates to achieve an optimal solution efficiently. Quadratic approximation is central to the formulation of SQP. Since nonlinear optimisation problems are inherently difficult to solve, SQP transforms them into a sequence of quadratic programming (QP) subproblems (Stoer, 1985). These subproblems are easier to solve and provide search directions toward the optimal solution. Lagrange multipliers play a crucial role in handling constraints effectively within SQP (Borzi, 2023). These multipliers provide sensitivity information about the objective function with respect to the constraints. Their presence in the Lagrangian function allows for an efficient representation of the optimisation problem that incorporates both the primal and dual aspects of the constraints.

SQP is rooted in nonlinear programming and optimisation theory, where the goal is to minimize or maximize an objective function subject to constraints. Given a general NLP problem:

$$\min_{x \in R^n} f(x) \qquad 1$$

subject to:

$$h_i(x) = 0, i = 1, \ldots, m \qquad 2$$

$$g_j(x) \leq 0, j = 1, \ldots, p \qquad 3$$

where:

$f(x)$ is the objective function,

$h_i(x)$represents equality constraints,

$g_j$represents inequality constraints.

SQP iteratively solves a quadratic subproblem to approximate the nonlinear objective function and constraints. The optimisation problem at each iteration is formulated using a second-order Taylor series expansion of the Lagrangian function.

3.2.1 The Lagrangian Function

The Lagrangian function for the NLP problem is given by:

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{j=1}^{p} \mu_j g_j(x) \qquad 4$$

Where hi are the Lagrange multipliers for the equality and inequality constraints, respectively.

3.3.2 Quadratic Approximation

At each iteration kk, SQP solves the following Quadratic Programming (QP) subproblem:

$$\min_{d} \nabla f(x_k)^T d + \frac{1}{2} d^T H_k d \qquad 5$$

Subject to:

$$h_i(x_k) + \nabla h_i(x_k)^T d = 0, \quad i = 1, \dots, m \qquad 6$$

$$g_j(x_k) + \nabla g_j(x_k)^T d \leq 0, \quad j = 1, \dots, p \qquad 7$$

Where:

d is the step direction,

H_k is an approximation of the Hessian matrix of the Lagrangian function, which ensures the quadratic nature of the problem.

The update step is:

$$x_{k+1} = x_k + \alpha_k d_k \qquad 8$$

Where α_k is a step size obtained using a line search or trust region method.

3.2.3 Key Components of SQP

3.2.3.1 Hessian Approximation

The Hessian matrix H_k can be computed using exact second derivatives (Newton's method) or approximated using quasi-Newton methods such as the Broyden–Fletcher–Goldfarb–Shanno (BFGS) update. The quasi-Newton update for $H_k$is given by:

$$H_{k+1} = H_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{H_k s_k s_k^T H_k}{s_k^T H_k s_k} \qquad 9$$

Where:

$$s_k = x_{k+1} - x_k \; and \; y_k = \nabla \mathcal{L}_{k+1} - \nabla \mathcal{L}_k. \qquad 10$$

3.2.3.2 KKT Conditions and Feasibility

SQP ensures convergence by satisfying the Karush-Kuhn-Tucker (KKT) conditions. At an optimal solution x^*, these conditions are:

Stationarity:

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla h_i(x^*) + \sum_{j=1}^{p} \mu_j^* \nabla g_j(x^*) = 0 \quad 11$$

Primal feasibility: $h_i(x^*) = 0, g_j(x^*) \leq 0$ 12

Dual feasibility: $\mu_j^* \geq 0$ 13

Complementary slackness:$\mu_j^* g_j(x^*) = 0$ 14

If these conditions are satisfied, the solution is optimal.

3.2.3.3 Constraint Handling

Inequality constraints in SQP are handled using active-set methods, where constraints that are active at the solution are treated as equalities in the QP subproblem. The algorithm dynamically updates the active set as iterations progress.

3.2.4. Convergence Properties of SQP

SQP has several desirable convergence properties:

3.2.4.1 Superlinear Convergence

When using an accurate Hessian approximation, SQP exhibits superlinear convergence, making it more efficient than first-order methods.

Robustness: SQP performs well even with tight constraints and nonlinearities.

3.2.4.1 Feasibility Preservation

Many SQP variants maintain feasibility at each step, ensuring practical applicability.

However, convergence can be affected by:

i. Poor Hessian approximations
ii. Ill-conditioned problems
iii. Constraint degeneracy

4.0 Formulation of SQP Model for Gas Lift Optimisation

There are various objective functions for gas lift optimisation using Sequential Quadratic Programming (SQP), however the three main ones include:

1. Maximising Oil Production Rate
2. Minimising Gas Injection Rate (Energy Consumption)
3. Maximising Gas Lift Efficiency

Each model includes decision variables, constraints, the Hessian matrix, and the SQP iterative update formulation.

Each objective function in gas lift optimisation using SQP follows a standard iterative procedure:

1. Formulate the objective function
2. Define constraints for production, gas injection, and system limits.
3. Construct the Lagrangian function incorporating constraints.
4. Compute gradients and Hessians to approximate the problem quadratically.
5. Solve the quadratic subproblem and update q_g iteratively.

4.1 Maximising Oil Production Rate

Objective Function:

$$\max_{q_g} q_o\left(q_g\right) \qquad 15$$

where: q_o = Oil production rate, q_g = Gas injection rate (decision variable)

The Mathematical Model is given as:

$$q_o = C_1 \cdot \left(q_g\right)^{C_2} \cdot \left(\frac{P_r - P_{wf}}{P_r}\right)^{C_3} \qquad 16$$

where: C_1, C_2, C_3 are empirical constants, P_r = Reservoir pressure, P_{wf} = Bottomhole flowing pressure, which depends on q_g

The constraints are:

i. Gas Injection Limits:

$$q_{g,min} \le q_g \le q_{g,max}$$

ii. Production Target:

$$q_o \ge q_{o,min}$$

iii. Wellbore Pressure Stability:

$$P_{wf,min} \le P_{wf} \le P_{wf,max}$$

SQP Formulation

The Lagrangian Function is given as:

$$\mathcal{L} = -q_o\left(q_g\right) + \sum_i \lambda_i g_i\left(q_g\right) \qquad 17$$

where $g_i\left(q_g\right)$ are constraint functions.

The Gradient of the Lagrangian (first derivative) becomes:

$$\nabla\mathcal{L} = -\frac{dq_o}{dq_g} + \sum_i \lambda_i \nabla g_i \qquad 18$$

The Hessian Matrix (second derivative) is given as:

$$H = \nabla^2\mathcal{L} = -\frac{d^2 q_o}{dq_g^2} + \sum_i \lambda_i \nabla^2 g_i \qquad 19$$

The Quadratic Approximation becomes:

$$\min_d \frac{1}{2} d^T H d + \nabla\mathcal{L}^T d \qquad 20$$

The update rule for q_g thus becomes:

$$q_g^{(k+1)} = q_g^{(k)} + \alpha d \qquad 21$$

where α is the step size.

4.2 Minimising Gas Injection Rate (Energy Consumption)

Objective Function

$$\min_{q_g} E\left(q_g\right) \qquad 22$$

Mathematical Model

The energy required for gas compression is:

$$E = \frac{ZRT}{\eta_{comp}}\left(\frac{P_{inj}}{P_s}\right)^{\gamma-1} q_g \qquad 23$$

where: Z = Compressibility factor, R = Gas constant, T = Temperature, η_{comp} = Compressor efficiency, γ = Gas adiabatic index, P_{inj} = Injection pressure, P_s = Standard gas pressure

Constraints

Maintain Oil Production Target: $q_o \ge q_{o,min}$

Gas Injection Limits: $x\, q_{g,min} \le q_g \le q_{g,max}$

Compressor Power Constraint: $\boldsymbol{E \le E_{max}}$

SQP Formulation

The Lagrangian Function is given as:

$$\mathcal{L} = E\left(q_g\right) + \sum_i \lambda_i g_i\left(q_g\right) \qquad 24$$

The Gradient of the Lagrangian becomes:

$$\nabla\mathcal{L} = \frac{dE}{dq_g} + \sum_i \lambda_i \nabla g_i \qquad 25$$

The hessian matrix is:

$$H = \nabla^2\mathcal{L} = \frac{d^2 E}{dq_g^2} + \sum_i \lambda_i \nabla^2 g_i \qquad 26$$

The Quadratic Approximation is:

$$\min_d \frac{1}{2} d^T H d + \nabla\mathcal{L}^T d \qquad 27$$

The Update Rule is:

$$q_g^{(k+1)} = q_g^{(k)} + \alpha d \qquad 29$$

4.3 Maximising Gas Lift Efficiency

The Objective Function is:

$$\max_{q_g} \eta = \frac{q_o(q_g)}{q_g} \qquad 30$$

The Mathematical Model is:

$$\eta\left(q_g\right) = \frac{C_1 \cdot \left(q_g\right)^{C_2} \cdot \left(\frac{P_r - P_{wf}}{P_r}\right)^{C_3}}{q_g} \qquad 31$$

The constraints is:

Production Constraint: $q q_o \ge q_{o,min}$

Gas Injection Limits: $\boldsymbol{q q_{g,min} \le q_g \le q_{g,max}}$

SQP Formulation

The Lagrangian Function is:

$$\mathcal{L} = -\eta\left(q_g\right) + \sum_i \lambda_i g_i\left(q_g\right) \qquad 32$$

The Gradient of the Lagrangian is:

$$\nabla\mathcal{L} = -\frac{d\eta}{dq_g} + \sum_i \lambda_i \nabla g_i \qquad 33$$

The Hessian Matrix is:

$$H = \nabla^2 \mathcal{L} = -\frac{d^2\eta}{dq_g^2} + \sum_i \lambda_i \nabla^2 g_i \quad 34$$

The Quadratic Approximation is:

$$\min_d \frac{1}{2} d^T H d + \nabla \mathcal{L}^T d \quad 35$$

The Update Rule is:

$$q_g^{(k+1)} = q_g^{(k)} + \alpha d \quad 36$$

5.0 Merits and Limitations of SQP in Gas Lift Optimisation

5.1 Merits of SQP in Gas Lift Optimisation

5.1.1. Robustness in Handling Nonlinearities and Constraint Management

One of the most compelling advantages of Sequential Quadratic Programming (SQP) in the realm of gas lift optimisation is its robustness and efficiency in handling complex nonlinearities and managing multiple constraints simultaneously. Gas lift optimisation inherently involves nonlinear relationships due to the physics of multiphase flow, the nonlinear behaviour of gas injection relative to oil production, and the complex interplay of pressure drops along the wellbore (Borzi, 2023). These nonlinearities stem from empirical correlations (such as Beggs-Brill or Hagedorn-Brown correlations) that are used to model the behaviour of the well. Additionally, numerous constraints—ranging from operational limits (maximum/minimum injection rates and pressures) to economic and safety constraints—must be rigorously satisfied.

5.1.2 Handling Nonlinear Objective Functions

Gas lift optimisation problems often have objective functions that are nonlinear, for example:

i. Maximising oil production rate: The production rate is a nonlinear function of the gas injection rate, tubing pressures, and other operational parameters.
ii. Minimising energy consumption: The energy required for gas compression or injection is a nonlinear function of gas properties and the operating conditions.
iii. Maximising gas lift efficiency: The ratio of oil produced per unit of gas injected is highly nonlinear.

SQP is well-suited for such problems because it approximates the original nonlinear problem by a series of quadratic subproblems. In each iteration, the method builds a quadratic model of the objective function and linear models of the constraints by using second-order derivative information (or approximations thereof). This approach allows the algorithm to capture the curvature of the objective landscape effectively, leading to more accurate and reliable convergence properties when compared to purely first-order methods (Borzi, 2023).

5.1.3 Efficient Constraint Handling

In gas lift optimisation, constraints are critical to ensuring safe and economically viable operations. These constraints include:

i. Physical constraints: Such as pressure limits at the wellbore and maximum allowable gas injection rates.
ii. Operational constraints: Ensuring that oil production meets a minimum threshold.
iii. Equipment constraints: For instance, limitations of compressors and gas lift valves.

SQP excels at handling such constraints by incorporating them directly into the quadratic programming subproblems. By formulating the Lagrangian function—an amalgamation of the objective function and the weighted sum of constraint violations—the algorithm adjusts its search direction not just to improve the objective but also to satisfy the constraints. This integrated approach is particularly useful in gas lift optimisation, where a minor violation of pressure constraints, for instance, could lead to unsafe operating conditions or damage to well integrity (Janka, 2015). Furthermore, the iterative update mechanism of SQP, which uses step-size adjustments and line-search techniques, ensures that each successive iterate remains feasible or is steered toward feasibility. This reliability is crucial in real-world gas lift operations where adherence to safety and regulatory constraints is non-negotiable.

5.1.4 Rapid Local Convergence

One notable feature of SQP is its rapid local convergence. Once the algorithm identifies a region close to an optimal solution, the use of second-order information (the Hessian matrix of the Lagrangian) allows SQP to converge quadratically. In the context of gas lift optimisation, where the operational environment may be subject to small but critical adjustments (for instance, slight variations in reservoir pressure or changes in gas properties), SQP's fast convergence ensures that optimal settings can be recalculated quickly in response to real-time data. This is particularly advantageous in fields that require adaptive control strategies, where the gas lift system must continually respond to evolving reservoir conditions (Janka, 2015).

5.1.5 Adaptability to Complex Models

Modern gas lift optimisation often involves coupling the well dynamics with reservoir models, which leads to highly complex and intertwined nonlinear models. SQP's framework is adaptable to such complexity. Since it only requires that the problem be twice differentiable (or at least approximations of the derivatives are available), SQP can be integrated with advanced simulation tools. Such complexity means that optimisation can be performed on the fly during simulation studies, allowing operators to test various scenarios and operational strategies before implementing them in the field (Borzi, 2023).

5.2 Limitations of SQP in Gas Lift Optimisation

5.2.1 Dependence on Accurate Derivative Information and Sensitivity to Problem Structure

While SQP offers numerous advantages, one of its significant disadvantages—particularly in the context of gas lift optimisation—is its heavy reliance on accurate derivative information and the sensitivity of its performance to the problem's structure. This dependence can manifest in various practical challenges that may undermine the robustness of the optimisation process, especially in the highly uncertain or noisy environments typical of oil field operations (Kungurtsev & Diehl, 2014).

5.2.2 Need for Accurate Derivative Computation

At the heart of the SQP method is the construction of quadratic models that require first- and second-order derivatives (gradients and Hessians) of the objective function and the constraints. For gas lift optimisation, obtaining these derivatives accurately can be challenging for several reasons:

i. Complex and Non-Smooth Models: The physical models used in gas lift optimisation, such as those describing multiphase flow dynamics, can be highly complex and sometimes exhibit non-smooth behaviour. Empirical correlations may not always be differentiable everywhere, or they might have regions where the derivative information is noisy. In such cases, the numerical approximation of derivatives may lead to inaccuracies that can mislead the SQP algorithm.
ii. Measurement Uncertainties: In a practical field setting, many parameters involved in gas lift optimisation (e.g., reservoir pressure, gas properties, and flow rates) are measured in real time and are subject to measurement errors and uncertainties. These uncertainties propagate in the derivative estimates. SQP’s performance is sensitive to such inaccuracies, which can cause convergence issues or lead the algorithm to settle at a suboptimal solution.
iii. Computational Expense: Calculating second-order derivatives, especially for complex systems, can be computationally expensive. In real-time or near-real-time optimisation scenarios, such as those required for adaptive control in gas lift operations, this computational burden might limit the speed of convergence. Although various quasi-Newton methods (like BFGS) can be used to approximate the Hessian matrix, the quality of these approximations is crucial for the success of SQP. Poor approximations can result in slow convergence or even divergence in the worst-case scenarios.

5.2.3 Sensitivity to Initial Guess and Local Optimality

SQP is a local optimisation method. This means that its convergence is highly dependent on the initial guess provided to the algorithm. In gas lift optimisation, the landscape of the objective function can be rugged, with multiple local optima, especially when the model includes non-convexities arising from complex fluid dynamics and operational constraints (Janka, 2015).

i. Local vs. Global Optimality: If the initial guess is not sufficiently close to the global optimum, SQP may converge to a local optimum that is significantly suboptimal from an operational standpoint. This is particularly problematic in gas lift operations, where small improvements in efficiency or production rates can have large economic implications over the lifetime of a well. The sensitivity of the starting point thus requires that operators invest additional effort in finding a good initial guess or use global optimisation techniques in tandem with SQP, which adds to the complexity of the overall optimisation process.
ii. Algorithmic Complexity: The iterative nature of SQP, which involves solving a quadratic subproblem at each iteration, means that its performance can degrade if the problem structure changes abruptly. For instance, if a sudden change in reservoir conditions causes the objective function's curvature to change dramatically, the previously computed Hessian (or its approximation) might become outdated. This necessitates recalculations or adjustments that can slow down convergence.

5.2.4 Robustness in the Face of Model Uncertainty

In the field of oil production, models are simplifications of reality. They might not capture all the nuances of the underlying physics, leading to model uncertainty. SQP's performance is directly affected by the accuracy of the model used. If the model does not accurately represent the true behaviour of the gas lift system, then the derivative data computed from it will be inaccurate. This misrepresentation can lead to poor optimisation results or unstable behaviour in the optimisation iterations (Kungurtsev & Diehl, 2014). To mitigate these issues, additional strategies, such as regularisation techniques or robust optimisation frameworks, must be integrated with SQP. This increases the complexity of the optimisation procedure, which may require further tuning and validation.

6.0 Scholarly Review of Application of SQP to Gas Lift

Wang *et al.* (2002) conducted a study on the optimisation of production operations in petroleum fields, focusing on the development of mathematical formulations for solving nonlinear constrained optimisation problems in gas lift networks. The research categorised the problem into two formulations, namely P1 and P2, and carried out a comparative analysis using Sequential Quadratic Programming (SQP) and Mixed-Integer Linear Programming (MILP) methods for gas lift optimisation. In Case 1, both Formulation P1 and Formulation P2 were solved using the SQP approach, and their performance was compared with MILP in terms of well rate and lift gas allocation optimisation. The findings indicated that SQP outperformed MILP due to its ability to account for the deliverability constraints of the gathering system, which MILP neglected by optimising well rates and lift gas rates based solely on gas lift performance curves. Case 2 evaluated the computational efficiency of Formulation P1 against Formulation P2, while Case 3 extended this analysis to test Formulation P2 on problems of varying sizes and complexities. The results demonstrated that the SQP method provided superior computational efficiency, leading to an 8% increase in field production rates while requiring significantly less injected gas. Additionally, the model was tested on multiple field scenarios, where it consistently proved to be robust and efficient, making it suitable for real-time production control and reservoir development.

Dehdari (2010) explored the application of SQP for solving constrained production optimisation problems, using a case study from the Brugge Field. The study focused on the development of equations governing the SQP framework, various formulations of SQP subproblems, and the enhancement of the optimisation algorithm. Key improvements included refining gradient estimation through covariance localisation, employing parallel computations, and updating QR factorisation. The results indicated that SQP significantly increased the rate of Net Present Value (NPV) growth compared to the steepest ascent method, an unconstrained optimisation algorithm. The study found that SQP improved cumulative oil recovery by 14% and achieved a considerably higher NPV compared to the steepest ascent method. Furthermore, the SQP algorithm exhibited lower sensitivity to initial conditions, demonstrating consistent performance across different seed numbers. Additionally, it was more effective in reducing water production, particularly in the later time steps of the optimisation process. The efficiency of the method was further enhanced by updating QR factorisation dynamically instead of recomputing it from scratch, which significantly reduced computational time. Parallel computing, using Message Passing Interface (MPI) for matrix multiplications and gradient calculations, also played a crucial role in accelerating the optimisation process. The elimination of non-negative constraints further reduced runtime, making SQP more applicable to large-scale optimisation problems despite the need for solving complex nonlinear equations.

Liu *et al.* (2018) conducted a comparative analysis of the Sequential Quadratic Programming (SQP) and Augmented Lagrangian (AL) algorithms for deterministic constrained production optimisation in hydrocarbon reservoirs. The study focused on optimising Net Present Value (NPV) for a three-phase reservoir under waterflooding conditions. The methodology included the development of the NPV function for a waterflooded reservoir, the formulation of a nonlinear constrained optimisation problem relative to NPV, and the implementation of the first-order optimality conditions, known as the Karush-Kuhn-Tucker (KKT) conditions. Further, the study involved the development of the Augmented Lagrangian (AL) approach and the formulation and solution of the SQP problem. The computational results showed that while the AL algorithm could achieve a slightly higher ultimate NPV under well-tuned parameters and favourable initial conditions, the SQP method demonstrated superior efficiency, robustness, and constraint-handling capabilities. Under SQP-optimised well control settings, the water cut in each producer adhered strictly to the imposed constraints, whereas the AL method led to minor constraint violations. The study also revealed that SQP achieved a 25% improvement in cumulative oil recovery. One major drawback of the AL approach was its sensitivity to initial conditions, particularly the penalty parameter, which affected the number of outer-loop iterations required for convergence. In contrast, SQP exhibited stable performance regardless of initial conditions and consistently achieved a higher NPV compared to AL when starting from an unfavourable initial point. To enhance performance, the study proposed a hybrid SQP-AL algorithm that applied the AL search direction to improve SQP convergence, leading to higher NPV with only a slight increase in iterations.

Kissoon *et al.* (2012) examined optimal gas utilisation strategies for maximising oil recovery in a mature oil field. The study methodology involved well model construction and validation, surface network modelling and optimisation, implementation of the SQP algorithm, optimisation of the surface network, and economic analysis of the optimisation results. In Case 1, an increase in oil production rate by 18% was observed, rising from 133 b/d to 162 b/d when a total of 4 MMscfd of gas was injected. Similarly, in Case 2, oil production increased by 15%, from 133 b/d to 154 b/d, when 3 MMscfd of gas was injected. The study concluded that optimal gas allocation to wells could lead to a 15% increase in oil production, a 30% reduction in gas utilisation, a 25% decrease in field operational costs, and a 42% increase in NPV. The integrated production model developed in this study proved useful for reservoir management, as it allowed real-time updates to production performance based on changes in water cut, reservoir pressure, and gas-oil ratio (GOR). The model also facilitated the generation of updated lift curves, enabling better reallocation of lift gas and the prediction of new oil rates and NPV values, making it a valuable tool for optimising mature field operations.

Yakoot *et al.* (2014) conducted a study on optimising gas-lift performance and multi-well networking in an Egyptian oil field using a simulation approach. The research investigated the effects of injection gas gravity and reservoir pressure on gas-lift performance and developed a total system production optimisation model using PROSPER and GAP software. The study focused on constructing well models using PROSPER, developing surface network models using GAP, implementing Sequential Quadratic Programming (SQP) under various constraints, optimising the surface network, and conducting economic analyses. The results revealed a steady increase in oil production as injection gas gravity increased, reaching an optimal value of 0.8, beyond which even minor increases led to significant production gains. The implementation of SQP within the GAP simulator resulted in an additional 200 barrels of oil per day while simultaneously reducing injection gas consumption by 5.5 MMScfd. This dual improvement in production and gas efficiency significantly enhanced the overall profitability of the field, demonstrating the potential of advanced optimisation techniques for field-wide gaslift management.

Diehl *et al.* (2018) explored strategies for increasing oil production in unstable gas-lift systems using nonlinear model predictive control. The study employed the transient multiphase flow simulator OLGA® and the dynamic process simulator UNISIM DESIGN® to simulate the oil production system under a nonlinear predictive control strategy. A comparative analysis of two nonlinear optimisation methods, the Local Linearisation Technique (LLT) and SQP, was conducted to evaluate their effectiveness. The methodology involved the development of ordinary and partial differential equations (ODEs and PDEs) to characterise the dynamic behaviour of oil wells, followed by the formulation and solution of optimisation problems using LLT and SQP algorithms. The study found that implementing a controller based on SQP led to a 45% increase in oil production, attributed to improved system stability and the ability to maintain optimal operating points. A significant portion of these gains resulted from shifting the operating point and stabilising intermittent flow, illustrating the power of nonlinear predictive control in enhancing production efficiency. The effectiveness of the optimisation was strongly linked to the reservoir flow constant Kr, which represents the well's productivity index, and the unstable equilibrium point, which determines the minimum achievable pressure for a specific choke opening and gas lift flow rate. The study concluded that SQP is a robust and efficient optimisation method for interconnected production networks, capable of significantly improving oil productivity in unstable gas-lift systems.

Okafor and Loyibo (2024) addressed the problem of nonlinear field network optimisation in gas-lift systems using SQP. The research developed nonlinear constrained equations for optimising gas-lift networks under limited gas allocation conditions. The study began with the construction of PROSPER models to simulate well-gas lift performance, incorporating key well parameters such as tubing size, water cut, bottom-hole flowing pressure (Pwf), and skin factor. These well models were then integrated into a GAP model to represent the surface network, linking PROSPER-generated vertical lift performance (VLP) curves with GAP to simulate production and injection networks. Two simulation scenarios were considered: in the first case, predefined gas injection rates were allocated to wells, and overall oil production was calculated; in the second case, GAP used SQP to dynamically allocate lift gas based on the optimal gas-lift potential of each well to maximise oil production. The results indicated that optimising the field production network using SQP in GAP, where each individual well was linked to a dedicated flowline, increased recovered oil by 1.5% and reduced total field lift gas injection rates by 2%. When production was optimised across four flowlines leading to a central manifold, oil production increased by 10%, with system efficiency improving by 20% and overall operational performance being significantly enhanced. The optimisation process demonstrated that a comprehensive field-wide gas-lift strategy could yield multiple benefits, including increased oil production, improved flow assurance, enhanced system efficiency, optimal gas allocation, and a reliable production plan. The study emphasised the importance of conducting gas-lift optimisation at a field-wide level to achieve holistic improvements in production network performance while addressing the nonlinear mathematical challenges associated with gas-lift allocation.

Sharma *et al.* (2012) investigated the nonlinear optimisation problem associated with the optimal distribution of lift gas among multiple oil wells. The study developed a nonlinear objective function based on a simplified dynamic model of an oil field, where the decision variables represented the lift gas flow rate set points for each oil well. The optimisation problem was solved using the 'fmincon' solver in MATLAB, with the results verified through the hill-climbing method. The study demonstrated that after optimisation, total oil production increased by approximately 4%. For fields where multiple oil wells share lift gas from a common source, a cascade control strategy combined with a nonlinear steady-state optimiser was found to function as a self-optimising control structure. This approach ensured that when the total supply of lift gas was the only input disturbance, repeated optimisation beyond the initial iteration had no further effect on total oil production. The findings highlighted the effectiveness of nonlinear optimisation in optimising lift gas distribution and improving field-wide production efficiency.

Dutta-Roy *et al.* (1997) presented a novel approach to the simulation and optimisation of the overall gas lift problem using a rigorous pressure-balance-based multiphase flow network technique coupled with a robust sequential quadratic programming (SQP) method for nonlinear constrained optimisation. The results of this new method were analysed and compared with conventional techniques, demonstrating a 0.56% increase in oil recovery when compared to traditional approaches.

Diehl *et al.* (2018) conducted a study where an oil production system was optimised using a nonlinear predictive control strategy. The evaluation of this strategy within a rigorous model (OLGA) highlighted the association between predictive capability and integrated actuation in the manipulated variables, leading to increased oil production and a partial or complete suppression of instabilities in multiphase flow. The control strategy significantly reduced the rate of valve actuation required, allowing for the use of slow choke valves as final control elements. Using a nonlinear predictive control strategy with a Flow-Orientated Well Model (FOWM) to predict dynamic behaviour near the bottom of the production column proved highly effective in improving the operation of unstable oil production systems. The results indicated a partial or total suppression of severe slugging and an increase in oil production. By manipulating the choke valve and gas lift flow, the nonlinear model predictive control (NMPC) could adjust the operating point of the well and find a more stable operational region while considering constraints such as limited gas availability in the gas lift system. Furthermore, NMPC was capable of reducing or eliminating oscillations in the naturally unstable open-loop region, stabilising the well at an average operating point comparable to the open-loop operation point. The controller functioned as a slugging attenuator, reducing instability intensity and increasing oil production. Part of this gain resulted from shifting the operating point, while another part was attributed to stabilising the intermittent flow. The study reported production gains of approximately 45%, which aligns with findings in previous research. However, the magnitude of these gains was strongly influenced by the reservoir flow constant (Kr), often called the well's productivity index, and the unstable equilibrium point, which represents the minimum pressure achievable for a given choke opening and gas lift flow rate. Consequently, the extent of oil productivity improvements depended on the characteristics of each production system. Regarding manipulated variables, the proposed control strategy offered two significant benefits: it enabled slow-acting choke valves to perform comparably to fast-acting ones, thereby broadening its implementation scope, and it balanced actuation intensity between the production choke and gas lift flow rate, leading to smoother variable adjustments. The ability to modify the operating point, enhance stability, and minimise control actions on manipulated variables made this approach highly advantageous for managing unstable gas-lifted wells.

Liu and Reynolds (2020) introduced a sequential-quadratic-programming-filter algorithm with a modified stochastic gradient for robust life-cycle optimisation problems involving nonlinear state constraints. They addressed an optimisation problem where the true gradients, which could not be computed analytically, were approximated using ensemble-based stochastic gradients through an improved stochastic simplex approximate gradient (StoSAG). Their study focused on waterflooding optimisation, with well controls as optimisation variables and the life-cycle net present value (NPV) of production as the cost function. The constrained optimisation problem was solved using SQP, with constraints enforced via the filter method. The researchers introduced modifications to StoSAG that improved its fidelity, yielding more accurate gradient approximations compared to the original algorithm. These improvements significantly enhanced the optimisation algorithm's performance. Without these modifications, applying SQP with the basic StoSAG approach could lead to highly suboptimal results, particularly for optimisation problems with nonlinear state constraints. Robust optimisation required each constraint to be satisfied across all reservoir models, posing a considerable computational challenge. While relying on reservoir simulations to enforce nonlinear state constraints heuristically reduced computational costs, it often led to inferior results. To address this, the researchers developed an alternative method for handling nonlinear state constraints that avoided explicit enforcement for each reservoir model while ensuring minimal constraint violations. This novel approach resulted in a 25% increase in cumulative field production, demonstrating its effectiveness in large-scale reservoir management.

Alarcón *et al.* (2002) presented a new mathematical fit for the gas-lift performance curve (GLPC), detailing numerical optimisation results and comparing them with other published methods. The GLPC could either be measured in the field or generated via computer simulations using nodal analysis. Their optimisation technique exhibited rapid convergence and broad applicability. Numerical experimentation confirmed the method's robustness, fast computation, and adherence to GLPC performance principles. The study revealed that while different optimisation techniques generally produced similar total oil flow rates (QoT), individual gas allocation and oil production varied significantly based on the chosen method. The accuracy of the GLPC data points played a crucial role in determining optimisation performance, underscoring the need to minimise uncertainties in field data and multiphase flow model predictions. The mathematical fit of the GLPC significantly influenced gas allocation for individual wells, with the proposed model yielding more reliable and accurate predictions due to its superior capability in fitting GLPC data points. The results indicated a 0.54% increase in cumulative field production using this optimised model, highlighting its potential for improving gas-lift allocation strategies. Overall, the research demonstrated that precise mathematical modelling of GLPC could enhance production efficiency and resource utilisation in gas-lifted oil fields.

Liu *et al.* (2018) conducted a study comparing the performance of sequential quadratic programming (SQP) and augmented Lagrangian (AL) algorithms for deterministic constrained production optimisation in hydrocarbon reservoirs. Both methods have been extensively utilised in solving nonlinear constrained optimisation problems, particularly in numerical optimisation. The study aimed to assess their robustness, efficiency, and ability to handle constraints within the context of optimising the PUNQ reservoir model. The gradients of the objective function and nonlinear constraints were estimated using the adjoint method, ensuring precise evaluations of sensitivities required for optimisation. The computational results indicated that with carefully tuned parameters and a well-chosen initial starting point, the AL method occasionally yielded a slightly higher net present value (NPV). However, in general, SQP exhibited superior performance in terms of efficiency, robustness, and constraint handling. One of the most significant advantages of SQP was its ability to strictly enforce production constraints, such as keeping the water cut for each producer within the specified limits. In contrast, the AL method, despite its effectiveness, occasionally resulted in minor violations of these constraints.

To further enhance optimisation performance, Liu *et al.* introduced an SQP-AL hybrid algorithm that incorporated the AL search direction within the SQP convergence process. The results demonstrated that this hybrid approach was capable of achieving a slightly higher NPV than the standalone SQP method. However, this improvement came at the cost of additional optimisation iterations and minor constraint violations at convergence. In the first case study, both SQP and AL were applied to the production optimisation problem under identical initial conditions. The AL algorithm required 128 simulations and 44 gradient evaluations to achieve convergence, whereas SQP reached convergence significantly faster, needing only 24 simulations and 14 gradient evaluations. These results highlighted the efficiency of SQP, making it a more computationally attractive choice.

The robustness of both algorithms was further examined by initiating optimisation from a suboptimal or "poor" starting point. The AL algorithm exhibited slower convergence, requiring additional outer-loop iterations and more simulations compared to cases with a well-chosen starting point. This inefficiency was attributed to the initialisation of the penalty parameter, which affected the algorithm's ability to quickly refine the optimisation trajectory. In contrast, SQP maintained stable performance regardless of the starting conditions and consistently achieved a higher ultimate NPV when initialised from a poor starting point.

A more stringent constraint scenario was also investigated, wherein the maximum allowable water cut for each production well was set to 0.8. Under these conditions, the SQP algorithm again demonstrated superior performance by efficiently managing the imposed constraints while maintaining robust optimisation performance. The AL algorithm, on the other hand, exhibited minor constraint violations at convergence, reinforcing the observation that SQP was more effective at handling highly constrained production optimisation problems. To further explore optimisation enhancements, Liu *et al.* implemented the hybrid SQP-AL approach in an additional case study. In this setup, the AL search direction was incorporated into the SQP framework during the final convergence phase. The results confirmed that the SQP-AL algorithm achieved a slightly higher NPV compared to the standalone SQP approach. However, this outcome came at the expense of increased optimisation iterations and occasional minor constraint violations. Despite these drawbacks, the hybrid methodology proved to be a viable alternative for maximising field performance.

The overall findings of the study underscore the advantages of SQP in terms of efficiency, robustness, and its ability to rigorously enforce constraints. While AL exhibited promising results in certain cases, its sensitivity to parameter tuning and starting conditions made it less reliable in practical applications. The introduction of the SQP-AL hybrid method further demonstrated the potential for improved optimisation outcomes, albeit with trade-offs in computational complexity. The study concluded that the application of these advanced optimisation techniques could significantly enhance hydrocarbon reservoir management, with the proposed optimisation model leading to an additional 16% increase in cumulative oil recovery from the field.

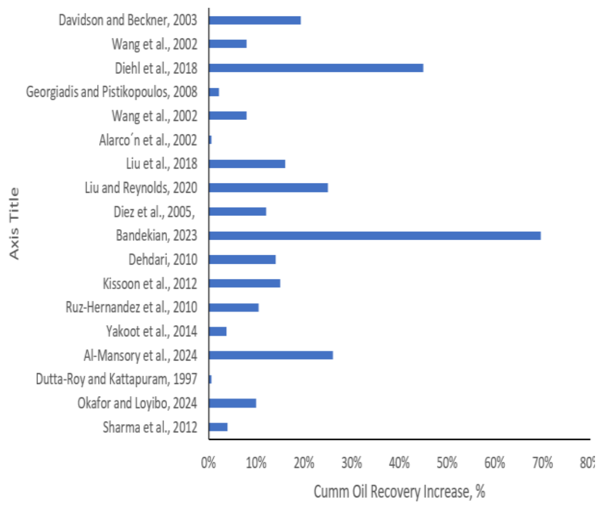
The survey on the use of SQP for production optimisation is presented in Table 2.

Table 2
Survey on the Use of SQP for Production Optimisation

S/No	Technique	Reservoir Fluid type	Well Type	Applicability	Parameter Analyzed	Difference	Reference
1	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	4%	Sharma *et al.* (2012)
2	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	10%	Okafor and Loyibo (2024)
3	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	0.56%	Dutta-Roy and Kattapuram (1997)
4	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	26%	Al-Mansory *et al.* 2024)
5	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	3.78%	Yakoot *et al.* (2014)
6	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	10.43%	Ruz-Hernandez et al. (2010)
7	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	15%	Kissoon *et al.* (2012)
8	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	14%	Dehdari (2010)
9	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	69.6%	Bandekian (2023)
10	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	12%	Diez *et al.* (2005)
11	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	25%	Liu and Reynolds, (2020)
12	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	16%	Liu *et al.* (2018)
13	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	0.54%	Alarco´n *et al.* (2002)
14	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production	8%	Wang *et al.* (2002)

					compared with MILP		
15	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	2.17%	Georgiadis and Pistikopoulos (2008)
16	SQP	Oil Reservoir fluid	Gas lifted well	Field network of wells	Cumulative Oil Production compared with MILP	45%	Diehl *et al.* (2018)
17	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	8%	Wang *et al.* (2002)
18	SQP	Oil Reservoir fluid	Gas Lifted well	Field network of wells	Cumulative Oil Production	19.35%	Davidson and Beckner (2003)

Figure 1
Plot of Increase in Cumulative Oil Recovery Due to Optimisation Using SQP By Scholars



SQP has proven to be a highly effective numerical optimisation technique for gas-lift optimisation, demonstrating superior computational efficiency, constraint-handling capabilities, and production enhancement compared to alternative methods. Its application in real-time production control, field-wide gas-lift management, and nonlinear model predictive control underscores its versatility in addressing complex petroleum engineering challenges. Continued advancements in hybrid optimisation strategies, machine learning integration, and computational efficiency will further enhance the applicability and impact of SQP in the oil and gas industry.

7.0 Conclusion

This study reviewed sequential quadratic programming (SQP) and its application to gas lift optimisation and underscored its efficacy as a robust numerical optimisation method capable of handling nonlinearly constrained problems with high accuracy and efficiency. SQP leverages second-order approximations of the objective function and constraints, utilising Hessian approximations and Lagrange multipliers to iteratively refine solutions. This ensures fast convergence and reliable performance, particularly for complex engineering applications such as gas lift optimisation, where operational constraints and nonlinearities pose significant challenges.

In gas lift optimisation, SQP provides a structured approach to maximising oil production by optimising injection gas allocation. The iterative nature of SQP enables the handling of the nonlinear interactions between gas injection rates, well productivity, and surface facility constraints, offering superior performance compared to traditional gradient-based and heuristic approaches. Moreover, SQP's ability to incorporate active constraint management makes it particularly suited for real-world applications where dynamic operational limits must be adhered to. The following conclusions were reached.

i. Superiority of SQP over other Methods: Across multiple studies, SQP consistently outperformed alternative optimisation techniques, including Mixed-Integer Linear Programming (MILP) and the Augmented Lagrangian (AL) method. SQP demonstrated superiority to MILP in accounting for the deliverability constraints of the gathering system, leading to enhanced optimisation of well rates and lift gas allocation. Similarly, SQP showed robustness in strictly enforcing production constraints, unlike AL, which was prone to minor constraint violations.
ii. Enhanced Computational Efficiency: The computational advantages of SQP were well-documented. SQP's faster

convergence in constrained production optimisation, attributing improvements to refined gradient estimation, parallel computing, and QR factorisation updates. The SQP-filter algorithm with a modified stochastic gradient approach significantly improved optimisation performance in large-scale reservoir management.

iii. Significant Gains in Oil Production and Economic Benefits: Various studies reported substantial improvements in production rates, Net Present Value (NPV), and gas efficiency. SQP showed as much as a 42% increase in NPV through optimal gas allocation. Applying SQP in nonlinear model predictive control led to a 45% increase in oil production by stabilising intermittent flow.

iv. Applicability to Large-Scale and Real-Time Optimisation: Studies showed SQP's ability to optimise large-scale gas-lift networks and dynamically allocate lift gas in real time. The integration of SQP into production simulation tools like PROSPER, GAP, and OLGA enabled robust field-wide optimisation strategies, leading to improved flow assurance and production stability.

v. Hybrid Optimisation Strategies: While SQP exhibited strong standalone performance, researchers explored hybrid approaches to further enhance optimisation outcomes.The SQP-AL hybrid algorithm leveraged AL search directions to improve SQP convergence, yielding higher NPV with minimal constraint violations. Similarly, combined nonlinear steady-state optimisation with cascade control strategies to optimise lift gas distribution efficiently.

vi. Consideration of Practical Constraints: One of the strengths of SQP is its ability to incorporate real-world constraints such as reservoir pressure limits, gas-lift performance curves, and economic factors. Many studies emphasised the importance of precise mathematical modelling in gas-lift performance curves, which significantly influenced gas allocation strategies and overall production efficiency.

8.0 Funding Statement

This research did not receive any external funding. It was basically done by the authors.

9.0 Acknowledgements

We sincerely acknowledge the Department of Petroleum Engineering in Rivers State University, the Department of Petroleum Engineering in Niger Delta University and all individuals who played a part in the accomplishment of this research project.

10. Declaration of Conflicting Interests

There was no conflict of interest.

8.0 Funding Statement

This research did not received any external funding. It was basically done by the authors.

9.0 Acknowledgments

We sincerely acknowledge the Department of Petroleum Engineering in Rivers State University, Department of Petroleum Engineering in Niger Delta University and all individuals who played a part in the accomplishment of this research project.

10. Declaration of Conflicting Interests

There was no conflict of interest.

11. References

Abdalsadig, M. A., Nourian, A., Nasr, G. G., & Babaie, M. (2016). Gas lift optimisation to improve well performance. *International Journal of Mechanical & Mechatronics Engineering*.

Adukwu, O., Odloak, D., & Junior, F. K. (2023). Gas-lifted systems: A hands-on tutorial on modelling and analysis. *Franklin Open, 4,* 100036.

Agwu, O. E., Alatefi, S., Azim, R. A., & Alkouh, A. (2024). Applications of artificial intelligence algorithms in artificial lift systems: A critical review. *Flow Measurement and Instrumentation,* 102613.

Ahmed, S. R., Sadeq, D. J., & Mahmud, H. B. (2023). Optimisation of gas lifting design in Mishrif formation of Halfaya oil field. *Iraqi Journal of Chemical and Petroleum Engineering, 24*(3), 45–54.

Alarcón, G. A., Torres, C. F., & Gómez, L. E. (2002). Global optimisation of gas allocation to a group of wells in artificial lift using nonlinear constrained programming. *Journal of Energy Resources Technology, 124*(4), 262–268.

Al-Mansory, S. H. O., Al-Fatlawi, O., & Kadkhodaie, A. (2024). Gas lift optimisation for Zubair oil field using genetic algorithm-based numerical simulation: Feasibility study. *Iraqi Journal of Chemical and Petroleum Engineering, 25*(2), 161–174.

Al-Swafi, N. I. (2022, July). Using of continuous gas lift process to increase productivity. In *AIP Conference Proceedings* (Vol. 2443, No. 1). AIP Publishing.

Avriel, M. (2020). Nonlinear programming. In *Mathematical Programming for Operations Researchers and Computer Scientists* (pp. 271–367). CRC Press.

Bahadori, A., & Zeidani, K. (2006). Compositional model improves gas-lift optimisation for Iranian oil field. *Oil & Gas Journal, 104*(5).

Bandekian, S. (2023). *Model-based gas lift optimisation for oil well networks using Python and PROSPER well models* (Master's thesis, NTNU).

Bomze, I. M., Demyanov, V. F., Fletcher, R., Terlaky, T., & Fletcher, R. (2010). The sequential quadratic programming method. In *Nonlinear optimisation: Lectures given at the CIME Summer School held in Cetraro, Italy, July 1–7, 2007* (pp. 165–214).

Borzì, A. (2023). *The sequential quadratic Hamiltonian method: Solving optimal control problems*. Chapman and Hall/CRC.

Curtis, F. E., & Overton, M. L. (2012). A sequential quadratic programming algorithm for nonconvex, nonsmooth constrained optimisation. *SIAM Journal on Optimisation, 22*(2), 474–500.

Davidson, J. E., & Beckner, B. L. (2003). Integrated optimisation for rate allocation in reservoir simulation. *SPE Reservoir Evaluation & Engineering, 6*(06), 426–432.

Dehdari, V., & Oliver, D. S. (2012). Sequential quadratic programming for solving constrained production optimisation: Case study from Brugge field. *SPE Journal, 17*(3), 874–884.

Dutta-Roy, K., & Kattapuram, J. (1997, June). A new approach to gas-lift allocation optimisation. In *SPE Western Regional Meeting* (pp. SPE-38333). SPE.

Guerra, L. A. O., Temer, B. O., Loureiro, J. B. R., & Freire, A. S. (2022). Experimental study of gas-lift systems with inclined gas jets. *Journal of Petroleum Science and Engineering, 216,* 110749.

Hannanu, M. I., Camponogara, E., Silva, T. L., & Hovd, M. (2024). A modified derivative-free SQP-filter trust-region method for uncertainty handling: Application in gas-lift optimisation. *Optimisation and Engineering,* 1–29.

Hepguler, G., Barua, S., & Bard, W. (1997). Integration of a field surface and production network with a reservoir simulator. *SPE Computer Applications, 9*(3), 88–92.

Ihua-Maduenyi, I. E., & Oguta, E. (2025). Numerical optimisation methods for enhancing oil production in gas lift systems: A review. *International Journal of Academic Multidisciplinary Research, 9*(3), 81–102.

Ihua-Maduenyi, I. E., & Yelebe, Z. R. (2025). Artificial intelligence techniques for gas lift network modeling and optimisation: A review. *International Journal of Academic and Applied Research, 9*(4), 131–152.

Janatian, N. (2024). *Real-time optimisation and control for oil production under uncertainty.*

Janka, D. (2015). *Sequential quadratic programming with indefinite Hessian approximations for nonlinear optimum experimental design for parameter estimation in differential–algebraic equations* (Doctoral dissertation).

Khishvand, M., & Khamehchi, E. (2012). Nonlinear risk optimisation approach to gas lift allocation optimisation. *Industrial & Engineering Chemistry Research, 51*(6), 2637–2643.

Kosmidis, V. D., Perkins, J. D., & Pistikopoulos, E. N. (2005). A mixed integer optimisation formulation for the well scheduling problem on petroleum fields. *Computers & Chemical Engineering, 29*(7), 1523–1541.

Krishnamoorthy, D., Fjalestad, K., & Skogestad, S. (2019). Optimal operation of oil and gas production using simple feedback control structures. *Control Engineering Practice, 91,* 104107.

Krishnamoorthy, D., Foss, B., & Skogestad, S. (2016). Real-time optimisation under uncertainty applied to a gas lifted well network. *Processes, 4*(4), 52.

Krishnamoorthy, D., Foss, B., & Skogestad, S. (2017). Gas lift optimisation under uncertainty. *Computer Aided Chemical Engineering, 40,* 1753–1758.

Krishnamoorthy, D., Jahanshahi, E., & Skogestad, S. (2018). Gas-lift optimisation by controlling marginal gas-oil ratio using transient measurements. *IFAC-PapersOnLine, 51*(8), 19–24.

Krishnamoorthy, D., Pavlov, A., & Li, Q. (2016). Robust extremum seeking control with application to gas lifted oil wells. *IFAC-PapersOnLine, 49*(13), 205–210.

Kungurtsev, V., & Diehl, M. (2014). Sequential quadratic programming methods for parametric nonlinear optimisation. *Computational Optimisation and Applications, 59,* 475–509.

Liu, Z., Forouzanfar, F., & Zhao, Y. (2018). Comparison of SQP and AL algorithms for deterministic constrained production optimisation of hydrocarbon reservoirs. *Journal of Petroleum Science and Engineering, 171,* 542–557.

Lu, C., Zhang, R., & Liu, S. (2016). A 0-1 integer programming model and solving strategies for the slab storage problem. *International Journal of Production Research, 54*(8), 2366–2376.

Mahdiani, M. R., Khamehchi, E., & Suratgar, A. A. (2019). Using modern heuristic algorithms for optimal control of a gas lifted field. *Journal of Petroleum Science and Engineering, 183,* 106348.

Mahmudi, M., & Sadeghi, M. T. (2013). The optimisation of continuous gas lift process using an integrated compositional model. *Journal of Petroleum Science and Engineering, 108,* 321–327.

Mehregan, M. R., Mohaghar, A., & Esmaeili, A. (2016). Developing a mathematical model for optimising oil production using gas-lift technology. *Indian Journal of Science and Technology, 9*(5), 24–32.

Miresmaeili, S. O. H., Pourafshary, P., & Farahani, F. J. (2016). Development of a time-dependent economic method with start time consideration to optimise gas-lift allocation and scheduling. *International Journal of Oil, Gas and Coal Technology, 13*(1), 41–59.

Miresmaeili, S. O. H., Zoveidavianpoor, M., Jalilavi, M., Gerami, S., & Rajabi, A. (2019). An improved optimisation method in gas allocation for continuous flow gas-lift system. *Journal of Petroleum Science and Engineering, 172,* 819–830.

Monfared, M., & Helalizadeh, A. (2013). Simulation and gas allocation optimisation of gaslift system using genetic algorithm method in one of Iranian oil field. *Journal of Basic and Applied Scientific Research,* 732–738.

Namdar, H., & Shahmohammadi, M. A. (2019). Optimisation of production and lift-gas allocation to producing wells by a new developed GLPC correlation and a simple optimisation method. *Energy Sources, Part A: Recovery, Utilisation, and Environmental Effects, 41*(21), 2616–2630.

Noorbakhsh, A., & Khamehchi, E. (2020). Field production optimisation using sequential quadratic programming (SQP) algorithm in ESP-implemented wells: A comparison approach. *Journal of Petroleum Science and Technology, 10,* 54-63. https://doi.org/10.22078/jpst.2020.3962.1629

Obong, B., Adegoke, P., Osuji-Bells, S., Ogbonna, D., Salisu, H., Ekerenduh, O., & Adomokhai, S. (2022, August). An integrated approach to production optimisation in ageing gas lifted fields: Ikanto field experience. In *SPE Nigeria Annual International Conference and Exhibition* (p. D021S001R003). SPE.

Okafor, C. V., & Loyibo, E. (2024). Gas lift optimisation: Solution to non-linear field network problem using sequential quadratic programming technique. *International Journal of Innovative Science and Research Technology, 9*(1), 1581–1603.

Okorocha, I. T., Chinwuko, C. E., Mgbemena, C. O., Godfrey, O. C., & Mgbemena, C. E. (2022). Production optimisation using gas lift incorporated with artificial neural network. *UNIZIK Journal of Engineering and Applied Sciences, 21*(1), 842–858.

Okorocha, I. T., Chinwuko, C. E., Mgbemena, C. E., & Mgbemena, C. O. (2020). Gas lift optimisation in the oil and gas production process: A review of production challenges and optimisation strategies. *International Journal of Industrial Optimisation, 1*(2), 61–70.

Pacheco, B. M., Seman, L. O., & Camponogara, E. (2023). Deep-learning-based early fixing for gas-lifted oil production optimisation: Supervised and weakly-supervised approaches. *arXiv preprint* arXiv:2309.00197.

Rostamian, A., de Moraes, M. B., Schiozer, D. J., & Coelho, G. P. (2024). A survey on multi-objective, model-based, oil and gas field development optimisation: Current status and future directions. *Petroleum Science.*

Salehian, M., Sefat, M. H., & Muradov, K. (2021). Robust integrated optimisation of well placement and control under field production constraints. *Journal of Petroleum Science and Engineering, 205,* 108926.

Sharma, R., Fjalestad, K., & Glemmestad, B. (2012). Optimisation of lift gas allocation in a gas lifted oil field as non-linear optimisation problem.

Sreenivasan, H., Patel, J., Jain, D., Patel, S., Wilson, I., & Krishna, S. (2023). Optimisation of gas lift system for well performance improvement in Asmari formation: A techno-economic perspective. *Petroleum Research.*

Sreenivasan, H., Patel, J., Jain, D., Patel, S., Wilson, I., & Krishna, S. (2024). Optimisation of gas lift system for well performance improvement in Asmari formation: A techno-economic perspective. *Petroleum Research, 9*(1), 115–124.

Stoer, J. (1985). Principles of sequential quadratic programming methods for solving nonlinear programs. In *Computational Mathematical Programming* (pp. 165–207). Springer.

Tavakoli, R., Daryasafar, A., Mohammad, K., & Milad, B. (2017). Optimisation of gas lift allocation using different models. *Recent Advances in Petrochemical Science, 1*(2).

Tewari, P., & Agrawal, B. (2022). A study of linear programming technique. *International Journal of Statistics and Applied Mathematics,* 2456–1452.

Vazquez-Roman, R., & Palafox-Hernández, P. (2005, October). A new approach for continuous gas lift simulation and optimisation. In *SPE Annual Technical Conference and Exhibition* (pp. SPE-95949). SPE.

Wang, P., Aziz, K., & Litvak, M. L. (2002). Optimisation of production from mature fields. In *World Petroleum Congress* (pp. WPC-32152). WPC.

Wilson, R. C. (1962). Some observations on facility design.

Zeinilabedini, L., & Ameli, F. (2025). Optimisation of dynamic gas injection in artificial lift with limited gas using an integrated system. *Journal of Petroleum Research.*

Zhong, H., Zhao, C., Xu, Z., & Zheng, C. (2022). Economical optimum gas allocation model considering different types of gas-lift performance curves. *Energies, 15*(19), 6950.